Newtonian Gravity in Theories with Inverse Powers of R

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Abstract

We discuss theories with inverse powers of the Ricci scalar. We find the exact metric produced by point masses in these theories, and show that the gravitational force between static objects is consistent with experiments.

I. INTRODUCTION

Recent astrophysical measurements seem to indicate that we live in an universe dominated by dark energy. The nature of this energy is still unknown. It might be a cosmological constant, or quintessence.

It is also possible that these new observations actually indicate that gravity needs to be modified on very long distance scales. Several models that produce such modifications exist in the literature. We will be particularly concerned with the model proposed in [1], where the gravitational action was modified by the addition of inverse powers of the Ricci scalar.

Explicitly, these authors considered an action of the form

$$S = \int d^4x \sqrt{-g} \left(R - \frac{\mu^4}{R} \right) \tag{1}$$

(We shall sometimes refer to this as the 1/R theory). This action is clearly singular at $R = 0$, and accordingly, Minkowski space is not a solution of this model. This model does possess solutions which are de Sitter or Anti-de Sitter. Therefore, the effects of dark energy can be reproduced by this modification of gravity[1].

It would seem that if μ is sufficiently small, then the effects of the new term should be negligible except on cosmological length scales $r \sim \mu^{-1}$. However, this intuition has been challenged by a number of authors [2, 3, 4, 5, 6]. The claim in these papers is that, in fact, the above theory is equivalent to a scalar-tensor theory, where the scalar field couples gravitationally and has an extremely tiny mass $m \sim \mu$. Such a theory is ruled out on many grounds, for instance solar system measurements.

Here we shall look at some solutions of this theory, in a hope to better understand the issues. In particular, we will find the *exact* solution for the field of a point mass in this theory. Remarkably, the solution is identical to the solution for a point mass in general relativity with a cosmological constant. Given the solution, it is straightforward to find the force between point particles and show that it is consistent with experiments. We discuss the possible significance of this solution.

II. GRAVITATIONAL SOURCE OF POINT PARTICLES IN 1/R THEORY

Here we find the exact gravitational solution for a point mass in the 1/R theory. We shall use the solution to find the forces between point masses and show that they are consistent with experiment.

The action

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left(R - \frac{\mu^4}{R} \right) + S_m \tag{2}$$

(S_m is the matter action) has the equation of motion

$$\left(1 + \frac{\mu^4}{R^2} \right) R_{\mu\nu} - \frac{1}{2} \left(1 - \frac{\mu^4}{R^2} \right) R g_{\mu\nu} + \mu^4 \left[g_{\mu\nu} D^2 - D_{(\mu} D_{\nu)} \right] R^{-2} = \kappa^2 T_{\mu\nu} \tag{3}$$

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[1] For previous work on this subject, see [2, 3, 4, 5, 6, 7, 8, 9, 10, 11].

Away from the localized sources, the right hand side can be set to zero.
We will start by looking for solutions of the form $R_{\mu\nu} = \pm\Lambda g_{\mu\nu}$. We then find

$$\Lambda = \frac{\sqrt{3}}{4}\mu^2 \tag{4}$$

So we find that away from source terms, any solution satisfying $R_{\mu\nu} = \pm\frac{\sqrt{3}}{4}\mu^2 g_{\mu\nu}$ also satisfies the equation of motion (3). That is, any solution of Einstein's equations with a positive *or* negative cosmological constant is a solution of the 1/R theory. This means that we can immediately write down a large class of solutions to equation (3), just by looking at solutions of $R_{\mu\nu} = \pm\Lambda g_{\mu\nu}$.

The simplest such solutions are:

a) de Sitter space

$$ds^2 = -\left(1 - \frac{\Lambda r^2}{3}\right)dt^2 + \left(1 - \frac{\Lambda r^2}{3}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{5}$$

and b) Anti-de Sitter space

$$ds^2 = -\left(1 + \frac{\Lambda r^2}{3}\right)dt^2 + \left(1 + \frac{\Lambda r^2}{3}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{6}$$

They represent the two vacuum solutions of the 1/R theory.

The remarkable feature here is that we could write down these solutions just by looking at previously known solutions of general relativity (GR) with a cosmological constant. This may clearly be extended to localized sources. Away from the sources, we solve $R_{\mu\nu} = \pm\Lambda g_{\mu\nu}$ with several localized sources (the sign is fixed by the boundary conditions.) This is now a problem in standard GR, and can be addressed using standard methods. The solution of the GR problem will produce a solution of (3) with the same sources. This applies to diffuse and time-dependent sources as well.

For a point source, the procedure outlined above yields the Schwarzschild-de-Sitter and Schwarzschild-Anti-de Sitter solutions which are of the form

c) Schwarzschild-de Sitter (black holes in de Sitter space)

$$ds^2 = -\left(1 - \frac{2M}{r} - \frac{\Lambda r^2}{3}\right)dt^2 + \left(1 - \frac{2M}{r} - \frac{\Lambda r^2}{3}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{7}$$

and d) Schwarzschild- anti de Sitter (black holes in anti de Sitter space)

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{\Lambda r^2}{3}\right)dt^2 + \left(1 - \frac{2M}{r} + \frac{\Lambda r^2}{3}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{8}$$

Outside the horizon, these satisfy the equation $R_{\mu\nu} = \pm\Lambda g_{\mu\nu}$, and therefore also satisfy the equation (3). These solutions represent the field produced by a point mass in the 1/R theory. To determine the solution fully, we still have to choose de-Sitter or anti-de-Sitter boundary conditions.

III. DISCUSSION

Let us return to the original question: the force between two masses in the 1/R theory. Since we are comparing with experiments in the real world, we should presumably take de Sitter boundary conditions. The field of each mass is then the de Sitter-Schwarzschild solution (7). The gravitational force can then be determined by taking the Newtonian limit.

The important point is that as μ gets small, the de-Sitter-Schwarzschild solution reduces to the Schwarzschild solution, so we are assured that we recover the standard Newtonian force law in the $\mu \to 0$ limit. The corrections to Newtonian gravity will show up at distance scales when $\Lambda r^2 \sim GM/r$, i.e. $r \sim (GM/\Lambda)^{1/3}$. For solar masses, this scale is roughly 10^{17} m, much larger than the solar system. So the deviations from Newtonian gravity on the scale of the solar system are negligible.

This appears to show that at least static solutions to these equations are in agreement with Newtonian gravity. The question is whether there are other solutions which are not of this form. In particular, the equations of motion are quartic, and it may be that further data is needed to specify the field of a point mass.

It might seem that the boundary conditions plus regularity should suffice to specify the solution for a point mass. However, the existence of the equivalent scalar-tensor theory with a light scalar sheds doubt on this speculation. It would be interesting to explore these issues further. It would be particularly interesting to see if the requirement of regularity of the metric puts constraints on the solutions of the scalar-tensor theory.

IV. ACKNOWLEDGEMENTS

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